Location Games on Graphs

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Abstract

We consider 2-player location games played on graphs, where the vertices and the edges of the graph may have weights. The graph could be considered to be a physical street network with villages as vertices, the vertex weights being the number of people, possible customers for the players, living there, and the edge weights being the lengths of the corresponding roads. Each player places a fixed number of locations on some vertices. People may or may not go to some location (to shop), depending on the distance to the closest location, but if they do, they will always choose the closest location, no matter to which one of the 2 players it belongs. The payoff for each player is the total number of customers in his or her locations.

We consider several variants, depending on

- the underlying vertex-weighted and edge-weighted graph G,
- the number k of locations each player can place,
- whether the game is simultanous, or sequential with 2 rounds, or sequential with $2k$ rounds,
- A (non-increasing) function f describing the tolerance of possible customers to distance. Only $f(x)$ of the customers will go to a location if the closest location has a distance of x.
- The rule how ties—one person having several locations with closest distance to him or her—are broken. Does that person visit any of the locations randomly, or use a 50%-50% split between locations of the two players in that case?

Note that the simultaneous games are symmetric. The games are ususally not zero-sum, except in the case of $f(x) = 1$, where all customers always visit one of the locations.

Note also that there are questions of how well the resulting pattern of locations serves the population as a whole. How many people visit some of the locations? What is the average length to the closest location for those who do? What is the average length to the closest location over the whole population?

Location games have been initiated by H. Hotelling in 1929 [H1929], considering the case of location on a straight line. Later, location games for other geometrical patterns have been investigated. Location games on graphs have been investigated by V. Knoblauch in two papers in 1991 and 1995. However, her models considered differ from ours. In [K1991], location everywhere on the edges is allowed. In [K1991], location is allowed only on some specified vertices of the graph, and only functions f of the form $f(x) = 1$ for $x \leq m$, and $f(x) = 0$ otherwise, are considered. In this second

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model, Knoblauch could show that every simultaneous 2-person symmetric game is equivalent to such a simultaneous location game on a graph.

In this talk we are going to discuss the following topics:

- We will discuss the question of what kind of games result from simultaneous location games with a function $f(x) = 1$ (meaning that every person will go to the closest location). We will also discuss the existence and number of pure Nash equilibria of these games.
- We will have a closer look into the case of simultaneous games with a function $f(x) = 1$ on special graphs, like trees or chordal graphs.
- We will discuss the relation to so-called "'potential games"' [MS1996] for simlutaneous location games with one location to place for each player. We will also discuss the existence of pure Nash equilibria.
- Some examples of simultaneous location games without pure Nash equilibria will be presented.
- We will discuss the welfare for the public of the resulting locations in some cases, using the measures discussed above.
- We will also discuss some of the sequential versions.

References

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